MBS-01-2016 Seat No. ________

M. Phil. (Sem. II) (CBCS) Examination

April / May - 2018

Mathematics

(Topology - CMT - 20001)

(New Course)

Faculty Code : 01

Subject Code : 2016

Time : $1\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) There are five questions in this paper.
(2) Each question carries 14 marks.
(3) All questions are compulsory.

1 Fill in the blanks : (Each question carries two marks) **14**

(a) If $f : xX \to \mathbb{R}$ is a continuous function then $f^1(\{\ \})$ is a __________ set.

(b) If I is a Z–ideal which contains a prime ideal then I is a __________ ideal.

(c) Every __________ maximal ideal in $C^*(\mathbb{N})$ contains the function $j(n)$.

(d) In a normal space X every __________ subset is C–embedded in X.

(e) In $C(\mathbb{N})$ every ideal is a __________ ideal.

(f) If A and B are completely separated in X then A and B are contained in disjoint. __________ sets.

(g) The space of natural numnbers is __________ embedded in its Stone – Cech compactification.

2 Attempt any **two** of the following : **14**

(a) Prove that an ideal M is a maximal ideal if and only if $Z(M)$ is a Z–ultra filter.

(b) State and prove the necessary and sufficient condition under which a subspace S of X is C^*–embedded in X.

(c) (i) Give an example of an ideal in $C(X)$ which is a Z–ideal.

(ii) Prove for any ideal I, $Z^{-1}(Z(1))$ is a Z–ideal.

3 All are compulsory : **14**

(a) Let $I = \{f \in C(\mathbb{R}) : Z(f) \textit{ is a neibhbourhood of } 0\}$ **6**

Show that I is a Z–ideal and it is not the intersection of maximal ideals containing it.

(b) Give an example of an ideal in $C^*(\mathbb{N})$ which is not the intersection of any ideal of $C(\mathbb{N})$ with $C^*(\mathbb{N})$. **4**

(c) Suppose I is a Z–ideal which contains a prime ideal. Prove that I is a prime ideal in $C(X)$. **4**

OR

3 All are compulsory : **14**

(a) Lex X be a compact Hausdorff space. **6**

(i) Prove that the closure of an ideal I in $C(X)$ is an ideal in $C(X)$.

(ii) Prove that every maximal ideal in $C(X)$ is closed. [$C(X)$=the Banach algebra of all complex valued continuous function on X].

(b) Give an example of a subset of $\mathbb{R}$ which is not C–embedded in $\mathbb{R}$ **4**

(c) Prove that every prime ideal of $C(X)$ is contained in a unique maximal ideal of $C(X)$. **4**

4 Attempt any **two** of the following : **14**

(a) Prove that a space X is compact if and only if every maximal ideal in $C^*(X)$ is fixed.

(b) Suppose X is a denes subspace of T and X is C^{*}–embedded in T. Prove that

   (i) If Z_1 and Z_2 are disjoint zero sets in X then $Cl_T(Z_1)$ and $Cl_T(Z_2)$ are disjoint.

   (ii) If Z_1 and Z_2 are zero sets in X then $Cl_T(Z_1 \cap Z_2) = Cl_T(Z_1) \cap Cl_T(Z_2)$.

(c) Let $C(X)$ be the Banach algebra of all complex valued continuous functions defined on a compacet Hausdorff space X. Prove that there is a one-one correspondence between the non-empty closed subsets of X and closed ideals of $C(X)$.

5 Do as directed : (Each question carries two marks) **14**

(a) Give reasons why $\mathbb{R} - \{0\}$ is not a zeros set.

(b) Give a continuous function $f : \mathbb{R} \to \mathbb{R}$ for which $Z(f)$ is a countable infinite set.

(c) Suppose $f \in C(X)$ and $A = \left\{ x \in X / f(x) > \frac{1}{2} \right\}$. Is A a zero set? Give reason.

(d) Suppose $f(x) = x$ for all x in $\mathbb{R}$ Let 1 be the principal ideal generated by $f(x)$. GIve a function g in $C(X)$ such that $g \in Z^{-1}(Z(1))$ but $g \notin I$.

(e) Give the definition of a compactification of a space X and give characteristic property of the Stone–Cech compactification of X.

(f) Give an example of a free maximal ideal in $C(\mathbb{N})$.

(g) Give a continuous function $f : \mathbb{N} \to \mathbb{R}$ such that f cannot be extended to a contiuous function $g : \beta(\mathbb{N}) \to \mathbb{R}$.